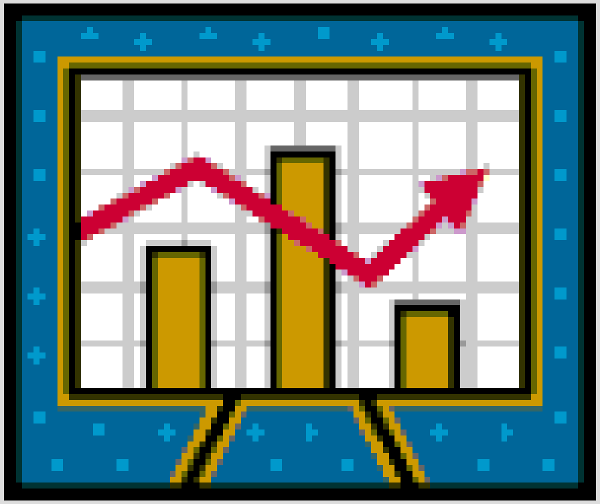
Managerial Economics & Business Strategy

Chapter 3

Quantitative Demand Analysis



Overview

I. The Elasticity Concept

- Own Price Elasticity
- Elasticity and Total Revenue
- Cross-Price Elasticity
- Income Elasticity

II. Demand Functions

- Linear
- Log-Linear

III. Regression Analysis

The Elasticity Concept

- How responsive is variable "G" to a change in variable "S"

$$E_{G,S} = \frac{\%\Delta G}{\%\Delta S}$$

If $E_{G,S} > 0$, then S and G are directly related.

If $E_{G,S} < 0$, then S and G are inversely related.

If $E_{G,S} = 0$, then S and G are unrelated.

The Elasticity Concept Using Calculus

- An alternative way to measure the elasticity of a function G = f(S) is

$$E_{G,S} = \frac{dG}{dS}\frac{S}{G}$$

If $E_{G,S} > 0$, then S and G are directly related.

If $E_{G,S} < 0$, then S and G are inversely related.

If $E_{G,S} = 0$, then S and G are unrelated.

Own Price Elasticity of Demand

$$E_{Q_X, P_X} = \frac{\%\Delta Q_X{}^d}{\%\Delta P_X}$$

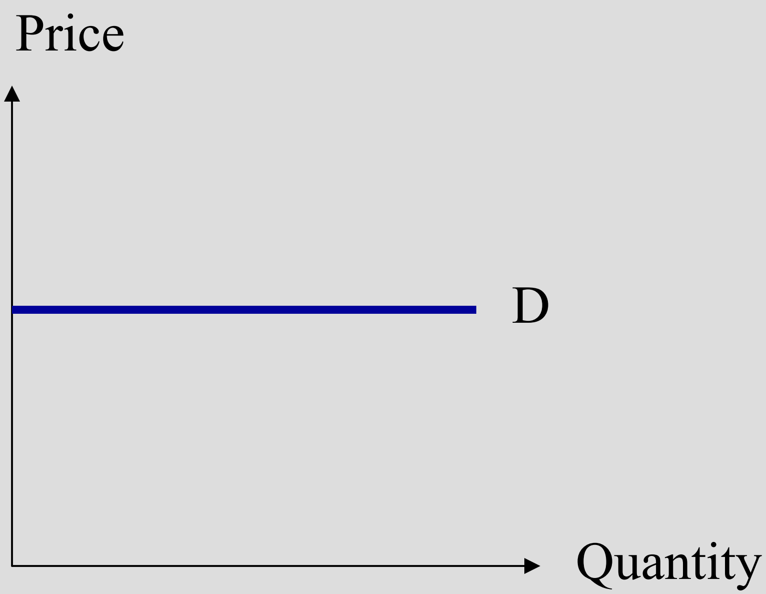
- Negative according to the "law of demand."

Elastic: $\left|E_{Q_X, P_X}\right| > 1$

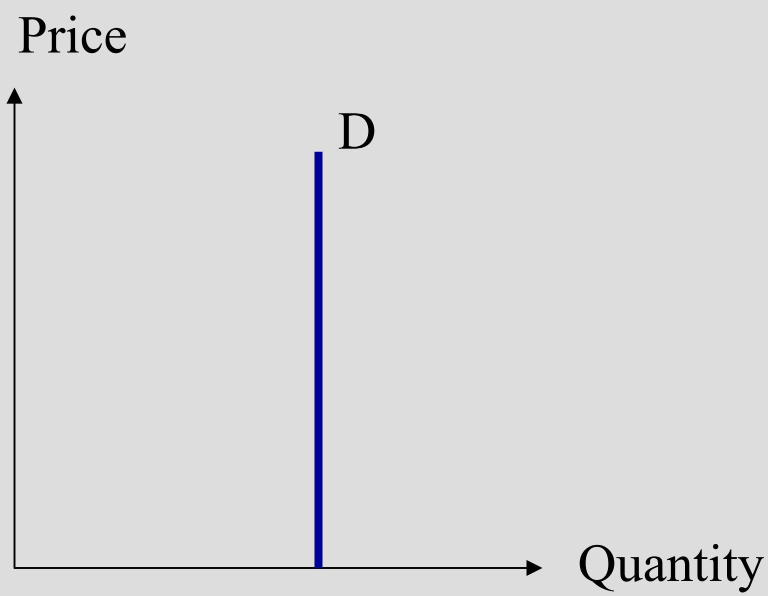
Inelastic: $\left|E_{Q_X, P_X}\right| < 1$

Unitary: $\left|E_{Q_X, P_X}\right| = 1$

Perfectly Elastic & Inelastic Demand



Perfectly Elastic ($E_{Q_X, P_X} = -\infty$)

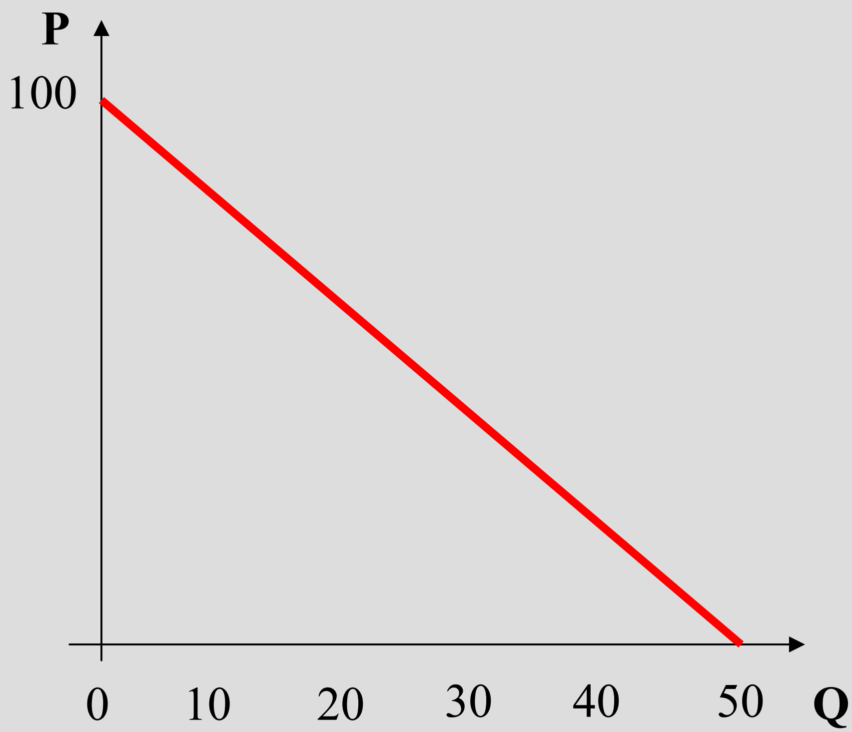


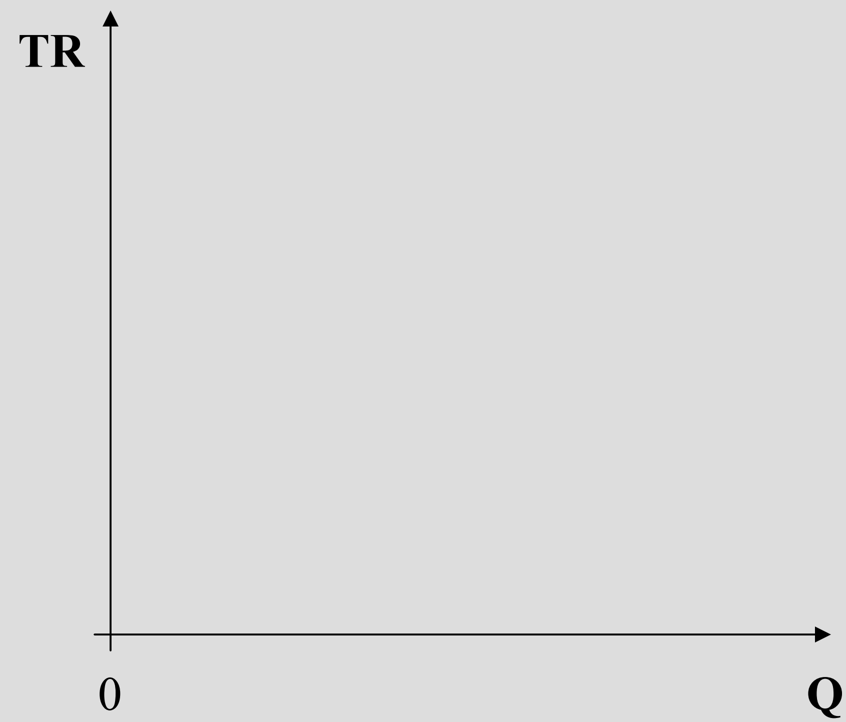
Perfectly Inelastic ($E_{Q_X, P_X} = 0$)

Own-Price Elasticity and Total Revenue

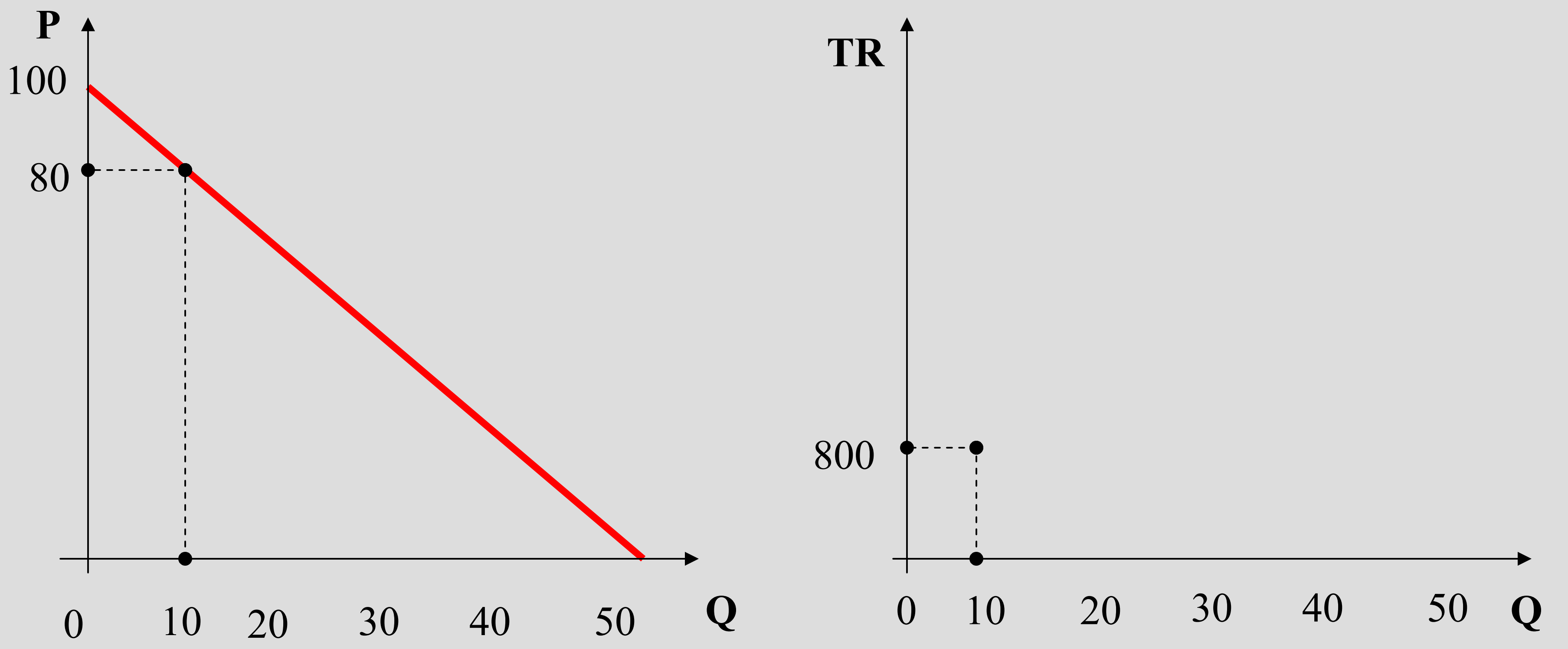
- Elastic
  - Increase (a decrease) in price leads to a decrease (an increase) in total revenue.
- Inelastic
  - Increase (a decrease) in price leads to an increase (a decrease) in total revenue.
- Unitary
  - Total revenue is maximized at the point where demand is unitary elastic.

Elasticity, Total Revenue and Linear Demand





Elasticity, Total Revenue and Linear Demand



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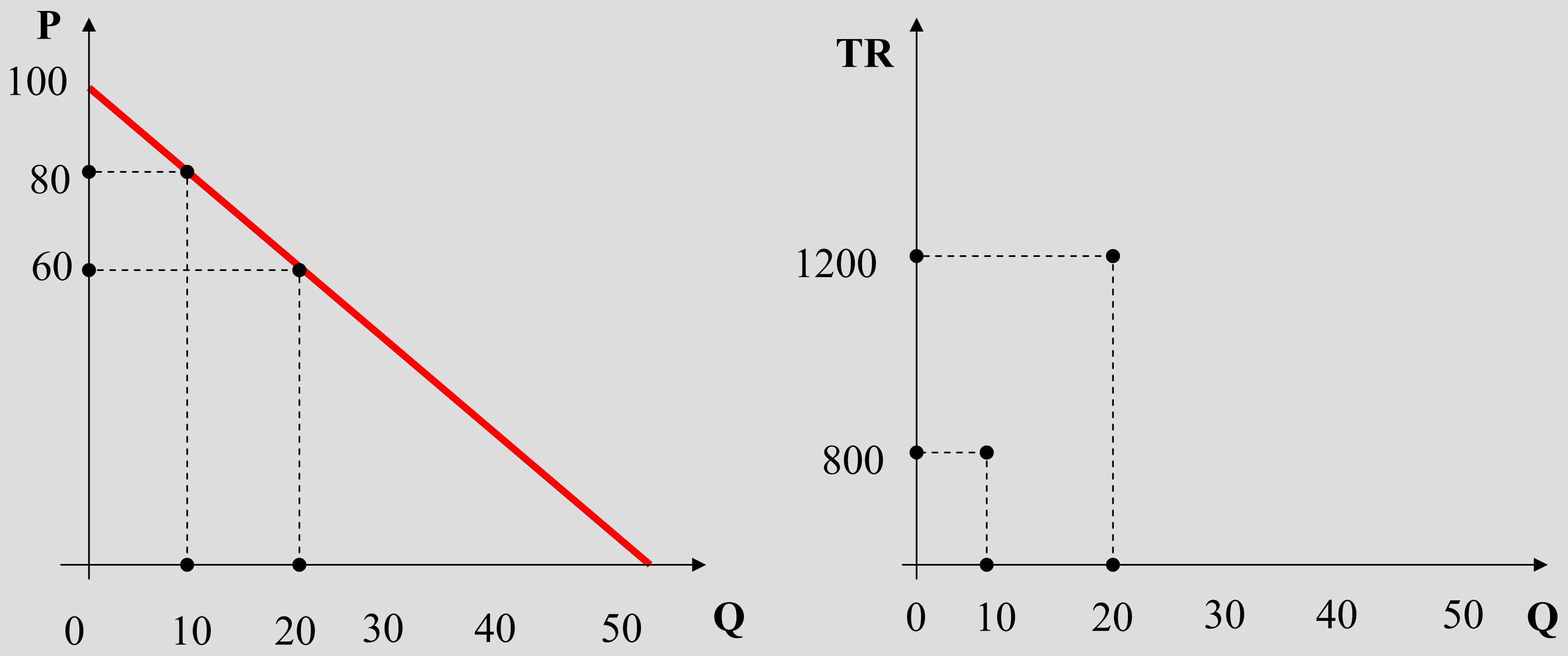
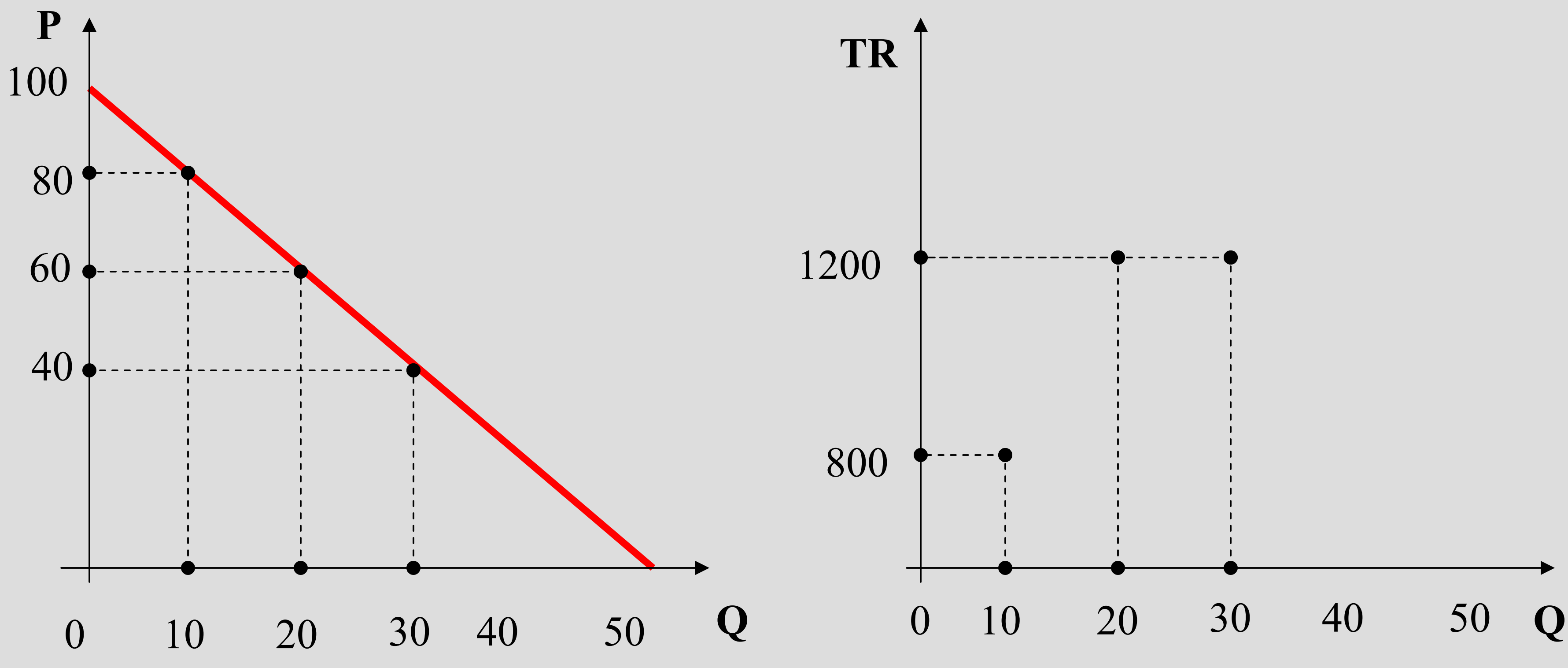
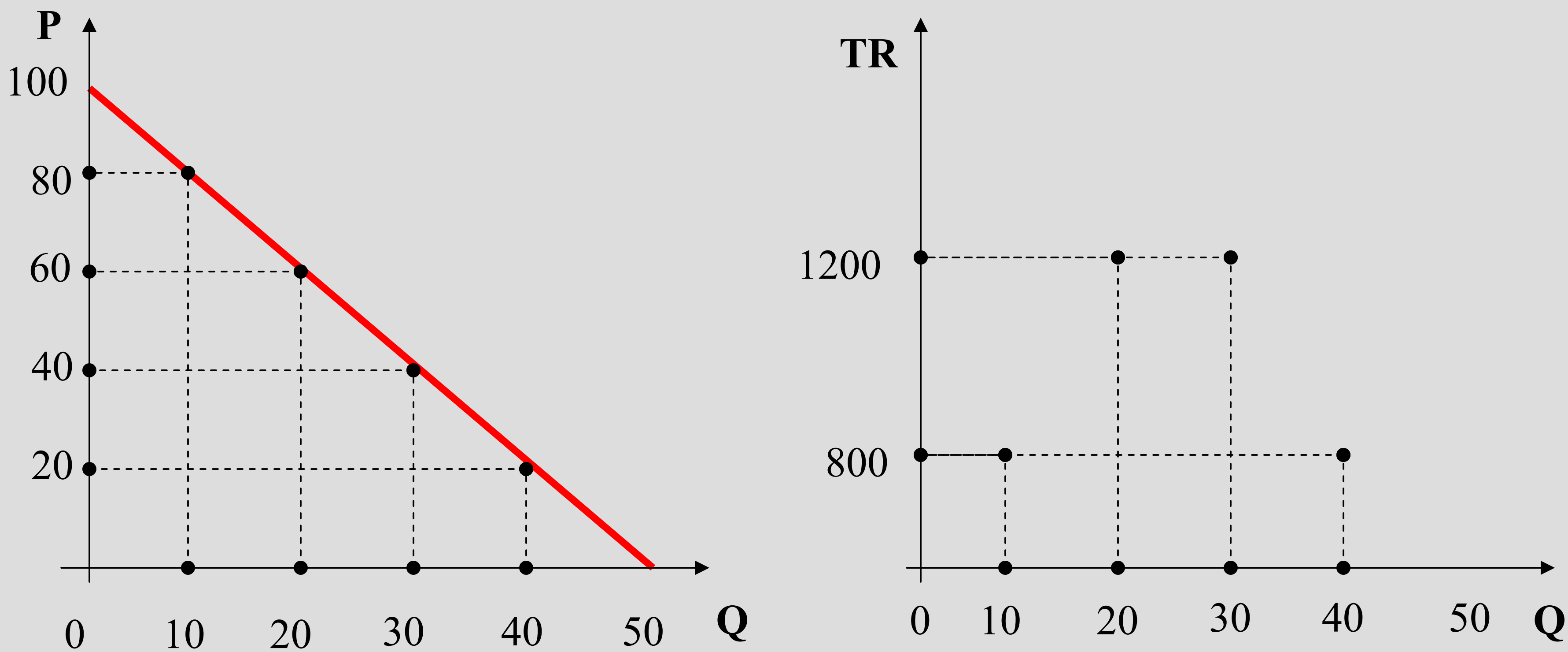


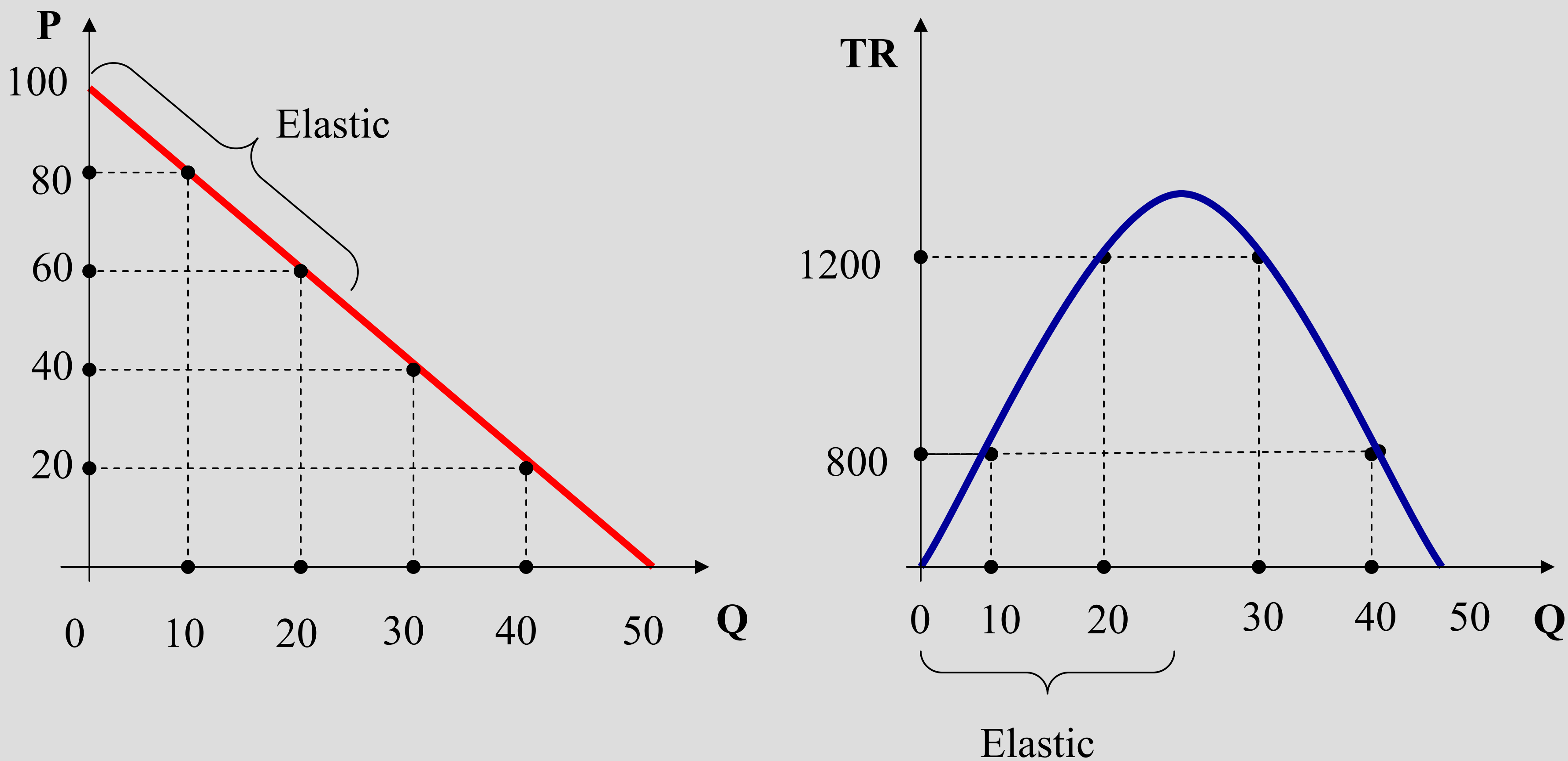
Elasticity, Total Revenue and Linear Demand



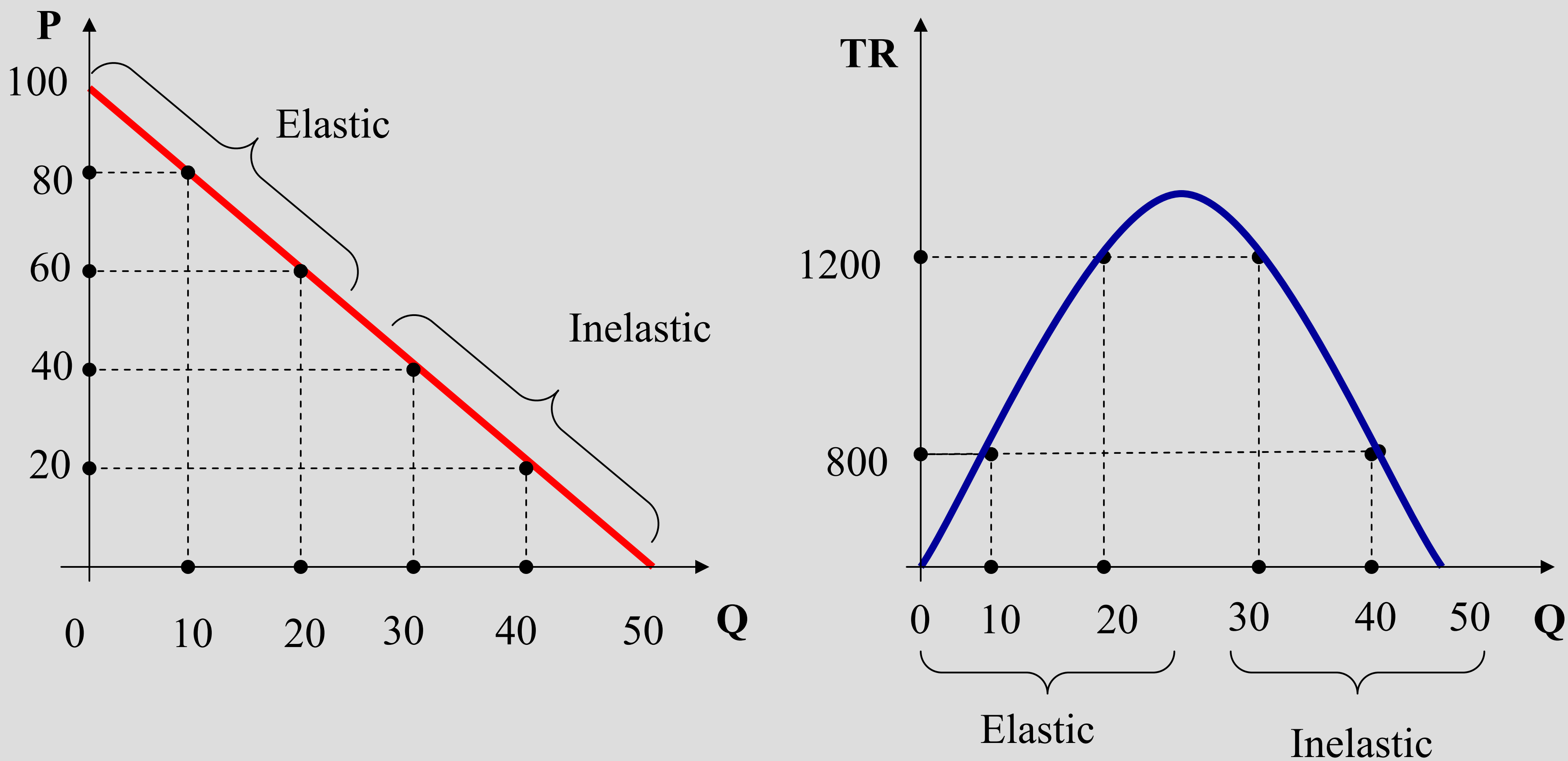
Elasticity, Total Revenue and Linear Demand



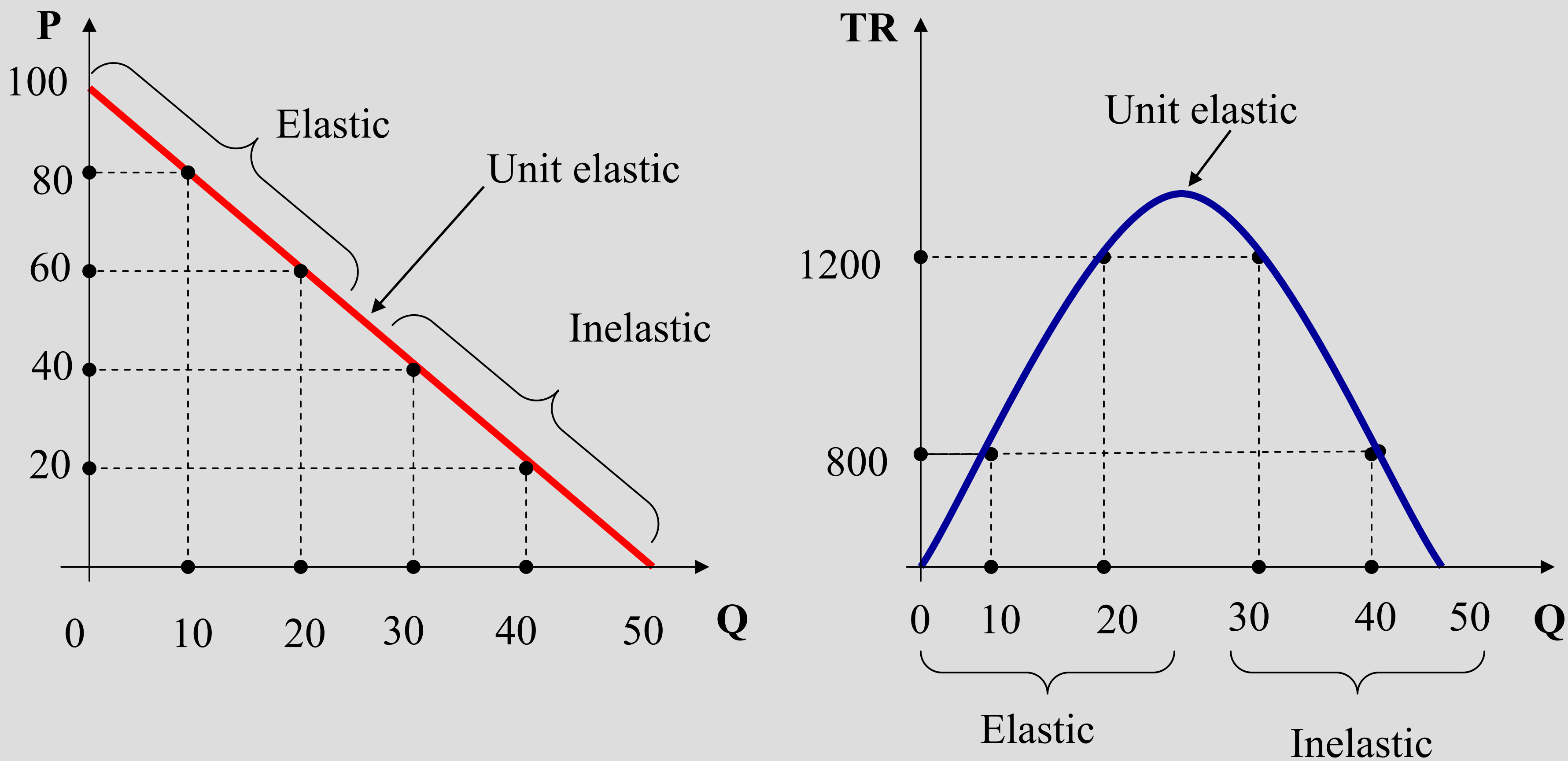
Elasticity, Total Revenue and Linear Demand



Elasticity, Total Revenue and Linear Demand



Elasticity, Total Revenue and Linear Demand



Factors Affecting Own Price Elasticity

- Available Substitutes
  - The more substitutes available for the good, the more elastic the demand.
- Time
  - Demand tends to be more inelastic in the short term than in the long term.
  - Time allows consumers to seek out available substitutes.
- Expenditure Share
  - Goods that comprise a small share of consumer's budgets tend to be more inelastic than goods for which consumers spend a large portion of their incomes.

Cross Price Elasticity of Demand

$$E_{Q_X,P_Y} = \frac{\%\Delta Q_X{}^d}{\%\Delta P_Y}$$

If $E_{Q_X,P_Y} > 0$, then X and Y are substitutes.

If $E_{Q_X,P_Y} < 0$, then X and Y are complements.

Predicting Revenue Changes from Two Products

Suppose that a firm sells to related goods. If the price of X changes, then total revenue will change by:

$$\Delta R = \left(R_X \left(1 + E_{Q_X, P_X}\right) + R_Y E_{Q_Y, P_X}\right) \times \%\Delta P_X$$

Income Elasticity

$$E_{Q_X,M} = \frac{\%\Delta Q_X{}^d}{\%\Delta M}$$

If $E_{Q_X,M} > 0$, then X is a normal good.

If $E_{Q_X,M} < 0$, then X is a inferior good.

Uses of Elasticities

- Pricing.
- Managing cash flows.
- Impact of changes in competitors’ prices.
- Impact of economic booms and recessions.
- Impact of advertising campaigns.
- And lots more!

Example 1: Pricing and Cash Flows

- According to an FTC Report by Michael Ward, AT&T's own price elasticity of demand for long distance services is -8.64.
- AT&T needs to boost revenues in order to meet it's marketing goals.
- To accomplish this goal, should AT&T raise or lower it's price?

Answer: Lower price!

- Since demand is elastic, a reduction in price will increase quantity demanded by a greater percentage than the price decline, resulting in more revenues for AT&T.

Example 2: Quantifying the Change

- If AT&T lowered price by 3 percent, what would happen to the volume of long distance telephone calls routed through AT&T?

Answer

- Calls would increase by 25.92 percent!

$$E_{Q_X, P_X} = -8.64 = \frac{\%\Delta Q_X^{\ d}}{\%\Delta P_X}$$

$$-8.64 = \frac{\%\Delta Q_X^{\ d}}{-3\%}$$

$$-3\% \times (-8.64) = \%\Delta Q_X^{\ d}$$

$$\%\Delta Q_X^{\ d} = 25.92\%$$

Example 3: Impact of a change in a competitor's price

- According to an FTC Report by Michael Ward, AT&T's cross price elasticity of demand for long distance services is 9.06.
- If competitors reduced their prices by 4 percent, what would happen to the demand for AT&T services?

Answer

- AT&T's demand would fall by 36.24 percent!

$$E_{Q_X, P_Y} = 9.06 = \frac{\%\Delta Q_X{}^d}{\%\Delta P_Y}$$

$$9.06 = \frac{\%\Delta Q_X{}^d}{-4\%}$$

$$-4\% \times 9.06 = \%\Delta Q_X{}^d$$

$$\%\Delta Q_X{}^d = -36.24\%$$

Interpreting Demand Functions

- Mathematical representations of demand curves.
- Example:

$$Q_X{}^d = 10 - 2P_X + 3P_Y - 2M$$

- X and Y are substitutes (coefficient of P_Y is positive).
- X is an inferior good (coefficient of M is negative).

Linear Demand Functions

- General Linear Demand Function:

$$Q_X{}^d = \alpha_0 + \alpha_X P_X + \alpha_Y P_Y + \alpha_M M + \alpha_H H$$

$$E_{Q_X,P_X} = \alpha_X \frac{P_X}{Q_X}$$

Own Price Elasticity

$$E_{Q_X,P_Y} = \alpha_Y \frac{P_Y}{Q_X}$$

Cross Price Elasticity

$$E_{Q_X,M} = \alpha_M \frac{M}{Q_X}$$

Income Elasticity

Example of Linear Demand

- $Q^d = 10 - 2P$.
- Own-Price Elasticity: $(-2)P/Q$.
- If P=1, Q=8 (since 10 - 2 = 8).
- Own price elasticity at P=1, Q=8:

  $(-2)(1)/8 = - 0.25$.

Log-Linear Demand

- General Log-Linear Demand Function:

$$\ln Q_X{}^d = \beta_0 + \beta_X \ln P_X + \beta_Y \ln P_Y + \beta_M \ln M + \beta_H \ln H$$

$$\text{Own Price Elasticity}: \quad \beta_{\text{X}}$$

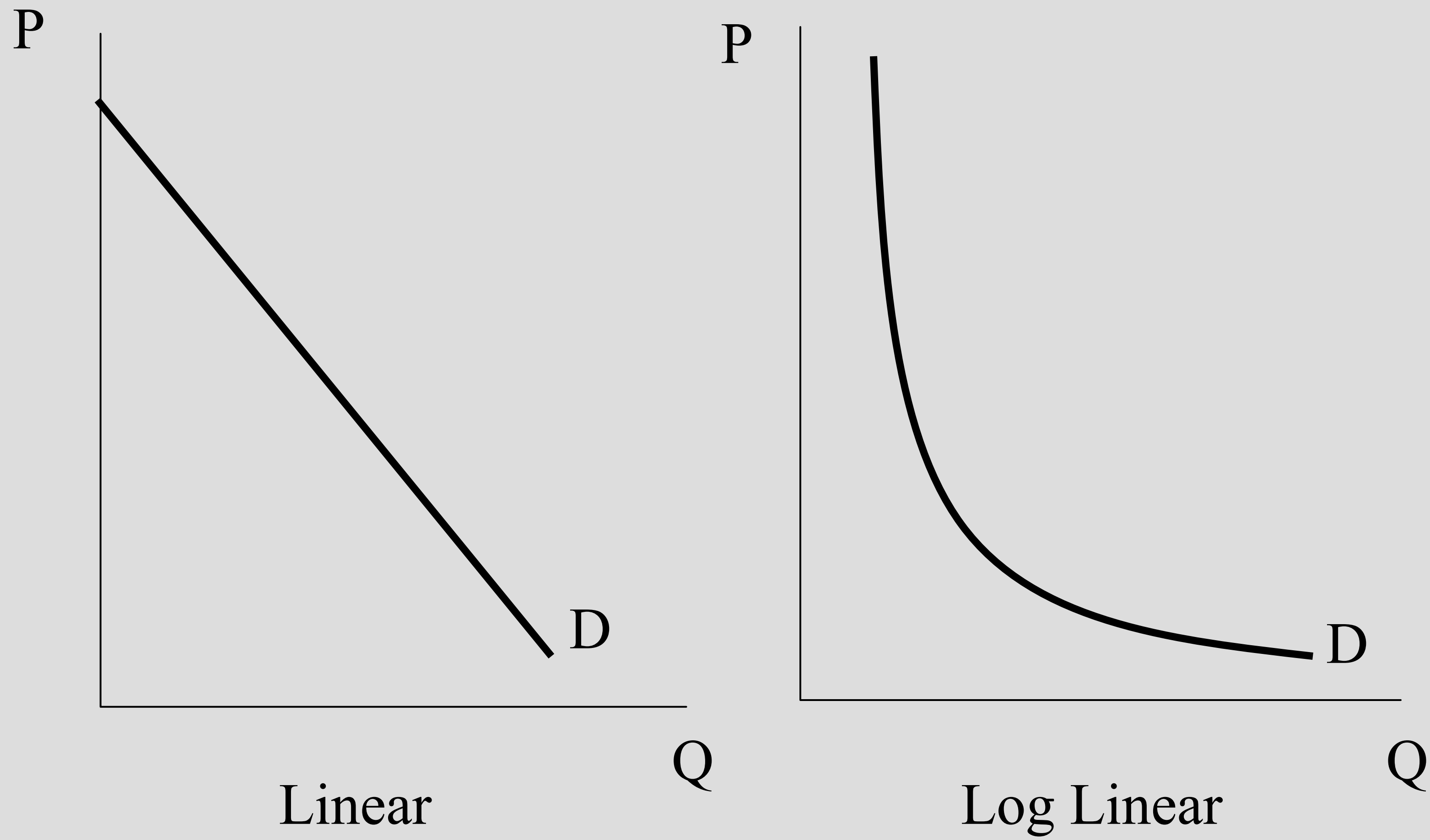
$$\text{Cross Price Elasticity}: \quad \beta_{\text{Y}}$$

$$\text{Income Elasticity}: \quad \beta_{\text{M}}$$

Example of Log-Linear Demand

- $\ln(Q^d) = 10 - 2 \ln(P)$.
- Own Price Elasticity: -2.

Graphical Representation of Linear and Log-Linear Demand



Regression Analysis

- One use is for estimating demand functions.
- Important terminology and concepts:
  - Least Squares Regression: Y = a + bX + e.
  - Confidence Intervals.
  - *t*-statistic.
  - *R*-square or Coefficient of Determination.
  - *F*-statistic.

An Example

- Use a spreadsheet to estimate the following log-linear demand function.

$$\ln Q_x = \beta_0 + \beta_x \ln P_x + e$$

Summary Output

Regression Statistics	
Multiple R	0.41
R Square	0.17
Adjusted R Square	0.15
Standard Error	0.68
Observations	41.00

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1.00	3.65	3.65	7.85	0.01
Residual	39.00	18.13	0.46		
Total	40.00	21.78			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	7.58	1.43	5.29	0.000005	4.68	10.48
ln(P)	-0.84	0.30	-2.80	0.007868	-1.44	-0.23

Interpreting the Regression Output

- The estimated log-linear demand function is:
  - $\ln(Q_x) = 7.58 - 0.84 \ln(P_x)$.
  - Own price elasticity: -0.84 (inelastic).
- How good is our estimate?
  - *t*-statistics of 5.29 and -2.80 indicate that the estimated coefficients are statistically different from zero.
  - *R*-square of .17 indicates we explained only 17 percent of the variation in $\ln(Q_x)$.
  - *F*-statistic significant at the 1 percent level.

Conclusion

- Elasticities are tools you can use to *quantify* the impact of changes in prices, income, and advertising on sales and revenues.
- Given market or survey data, regression analysis can be used to estimate:
  - Demand functions.
  - Elasticities.
  - A host of other things, including cost functions.
- Managers can quantify the impact of changes in prices, income, advertising, etc.